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FLOW THROUGH DIVERGING OPEN CHANNEL TRANSITIONS AT SUPERCRITICAL VELOCITIES

Progress Report Covering Tests with Straight
Flaring Sidewalls and 1 Percent Channel Slope

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FLOW THROUGH DIVERGING OPEN CHANNEL TRANSITIONS AT SUPERCRITICAL VELOCITIES

Progress Report Covering Tests with Straight Flaring Sidewalls and 1 Percent Channel Slope

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I. SYNOPSIS

Tests made on the SAF stilling basin have shown that for any given discharge the size of the basin can be reduced if the Froude number is increased. This can be accomplished by increasing the velocity or by decreasing the depth of flow. It is difficult to change the velocity in a relatively short distance, but the depth can be reduced if certain precautions are observed. However, it should be recognized that when water is flowing at high (supercritical) velocities, attempts to change the depth may result in poor and even dangerous flow conditions.

Experiments were initiated to determine the best means of reducing the depth of flow and to develop rules for the design of expanding transitions. Data and design rules are presented in this report for the case of transitions having straight flaring sidewalls located on a plane surface which is sloped just sufficiently to provide drainage.

II. NOMENCLATURE

Although the symbols are defined where they first appear, they are assembled below for the convenience of the reader.

A	wetted cross-sectional area
b	breadth
c	celerity of a gravity wave
d	depth of flow
D	sidewall divergence, one transverse to D longitudinal
$\underline{F}$	Froude number, v^2/gd (On figures $\underline{F}$ is written in bold-faced type.)
g	acceleration due to gravity
H	energy head, $v^2/2g + d$
S	slope of transition platform
v	velocity
v_c	critical velocity
v	nondimensional velocity, v/v_c
x	longitudinal distance from beginning of transition
y	transverse distance from transition centerline
β	wave angle, $\sin^{-1}\frac{1}{\sqrt{\underline{F}}}$, $\sin^{-1}\frac{c}{v}$
θ	angle between original and deflected velocity direction

Subscript "1" refers to initial flow conditions.

III. REASON FOR TESTS

The tests described here were made primarily to provide data for the design of a transition to be used between an open channel and a SAF stilling basin.[1] Preliminary computations had shown that an increase in the Froude number at the stilling basin entrance would permit smaller basins to be used. These computations indicated that the saving in the cost of the basin would more than offset the cost of installing the transition. The magnitude of this saving is discussed in Section IX, "Use of Transition with SAF Stilling Basin," page 18. In addition, more energy is dissipated and the basin functions more efficiently at high Froude numbers.

The Froude number $\underline{F}$, which will be used throughout this paper, is defined as

$$\underline{F} = \frac{v^2}{gd} \qquad (1)$$

where v is the velocity, g is the gravitational constant, and d is the depth of flow.

It is apparent that the local Froude number will vary along a transition as the depth and velocity change. The Froude number in the approach channel $\underline{F}_1$ will ordinarily be computed from the normal velocity and depth in that channel. Local velocities and depths will be used to compute $\underline{F}$ at any desired cross section along the transition. When stilling basins are designed, $\underline{F}$ is computed from the velocity and depth existing at the downstream end of the transition. In other words, at the entrance to the stilling basin.

The balance between the velocity and the losses in the approach channel is disturbed when a transition is used and must be reestablished. However, a much greater distance is required to establish normal velocity than is ordinarily available in transitions such as might be used at the entrances to SAF stilling basins. Therefore, the velocity can be considered constant for all practical purposes. Since v and g are constant, the depth must be decreased if the Froude number is to be increased. This can be accomplished by spreading the water in a diverging transition.

The design of transitions for subcritical velocities $(\underline{F} < 1)$ is relatively simple as compared to the design of transitions for supercritical velocities $(\underline{F} > 1)$. The performances of transitions designed for low velocities have been observed over the years, and what can or cannot be done is now well established. However, this statement becomes invalid when the velocity approaches or exceeds the critical. In fact, the literature on flow through transitions at supercritical velocities leads one to believe that no work whatever was done prior to the 1930's. Attempts to change the direction of water flowing at supercritical velocities frequently run into unexpected difficulties which are evidenced by standing waves. These waves originate at a change or discontinuity in the boundary. They travel across the stream at an angle, and where they hit the sidewalls may cause overtopping since their height can be twice the mean depth of flow. An example is shown in figure 1, page 3. Fortunately, no damage is caused by the slight overtopping at St. Anthony Falls, but here is a potential source of trouble that should be considered when the velocities are supercritical.

The reasons given above explain why these transition tests were initiated. They are:

(1) A more efficient and more economical SAF stilling basin can be used; and

(2) Design methods had not been developed which would insure satisfactory performance in the completed structure when the velocities are supercritical.

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[1] BLAISDELL, F. W. The SAF Stilling Basin. U. S. Dept. Agr., Soil Conserv. Serv. 17 p., illus. 1943. (Mimeographed.)

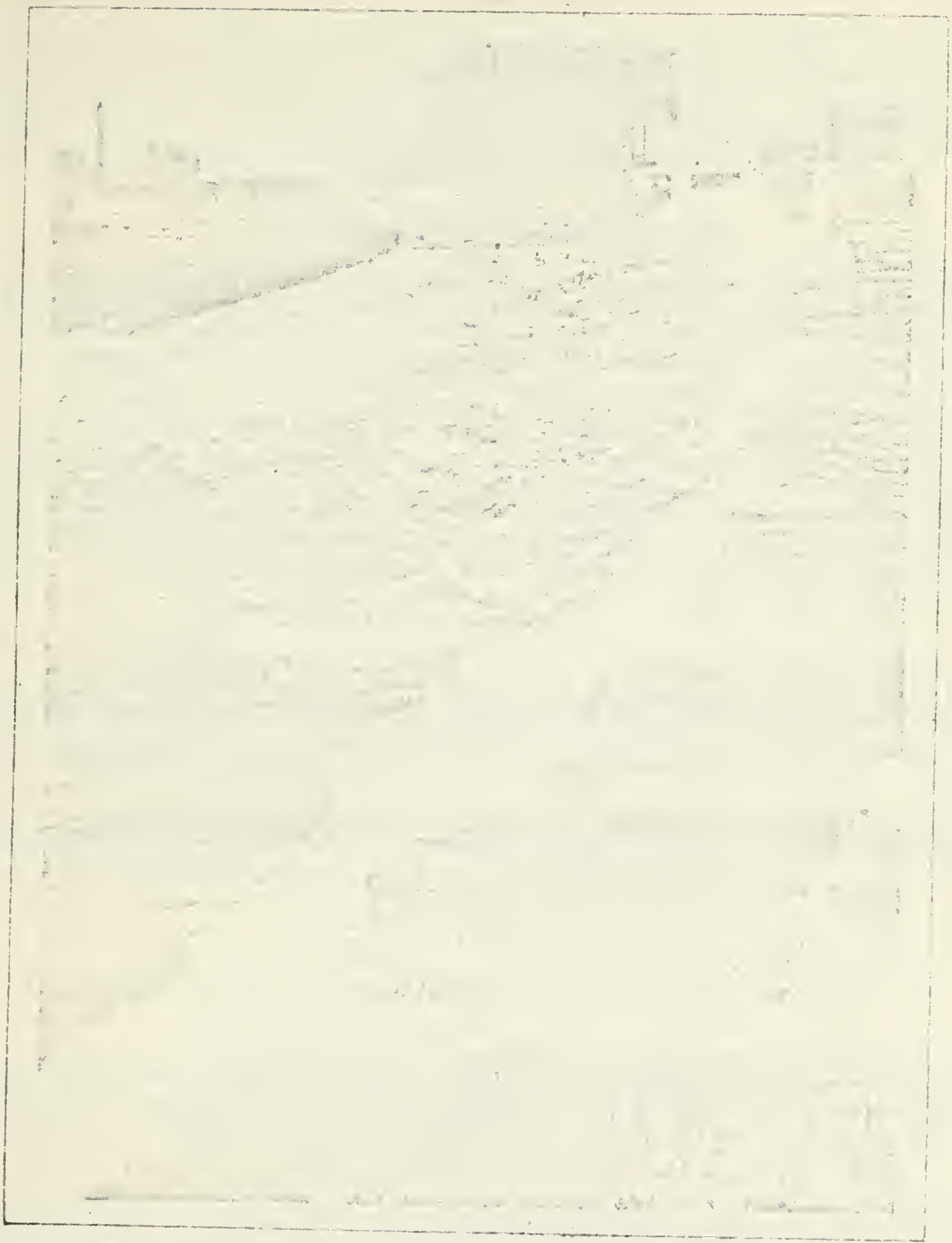

FIGURE 1.--Standing waves on floodway at St. Anthony Falls, Minneapolis, Minn.

Flow from the two open gates spreads out on the ledge upstream from the chute and is reflected from each sidewall as standing waves. The reflected waves cross at the head of the chute, causing a high crest, and continue on. The wave originating at the far wall strikes, overtops, and is reflected from the chute wall in the foreground.

The work reported here was initiated in 1944, and at present (1947) the study is continuing on a part-time basis. The tests were performed by the writer at the St. Anthony Falls Hydraulic Laboratory of the University of Minnesota at Minneapolis where the Minnesota Agricultural Experiment Station, the St. Anthony Falls Hydraulic Laboratory, and the Soil Conservation Service cooperate in the solution of problems in conservation hydraulics under the direction of the writer as Project Supervisor. Dr. M. L. Nichols is Chief of Research of the Soil Conservation Service, and Mr. Lewis A. Jones is Chief of its Division of Drainage and Water Control, of which the project at Minneapolis is a part.

IV. THEORETICAL CONSIDERATIONS

The theory of flow of water through transitions at supercritical velocities is a recent development that has been adapted from the analogous theory of flow of gases at supersonic velocities. The theory as applied to gases was mentioned by L. Prandtl in 1907 or earlier, but it was not until 1920 that Émile Jouguet pointed out the analogy between gases and water. In 1938 Theodor von Kármán[2] published a differential equation for the change in water surface elevation across a disturbance line[3] and integrated this equation along a curved sidewall for the region prior to the beginning of mutual influence of both curved walls. Von Kármán's integrated equations are not adapted to practical use because of the tedious work required to evaluate them and the limited range of their applicability. A completely general graphical solution of the basic differential equation was published in 1929 by L. Prandtl and A. Busemann for problems involving gaseous flows and was adapted to water flows by Ernst Preiswerk.[4] This graphical solution is not difficult and has proved to be of considerable value during the study discussed in this paper.

A. Basic Considerations

The theory of flow in open channel transitions will not be developed here. It has been ably presented by others.[5] Nevertheless, it may be well to explain how and why the depth changes occur since they occur differently when the velocity is greater than the critical than they do for the more familiar case where the velocity is less than the critical.

The wave pattern produced by dropping a pebble into still water is familiar to everyone. As shown in figure 2(a), page 5, these waves travel away from the source of the disturbance in all directions with a velocity or celerity c. In the case of a gravity wave

$$c = \sqrt{gd} \tag{2}$$

where d is the depth of water.

If the water is flowing, the wave pattern produced by a disturbance will be displaced in the direction of the flow. When the velocity v of the water is equal to the celerity, then the wave fronts formerly traveling upstream remain stationary, and those formerly traveling downstream at a celerity c have their velocity increased to c + v. This is shown in figure 2(b). At this

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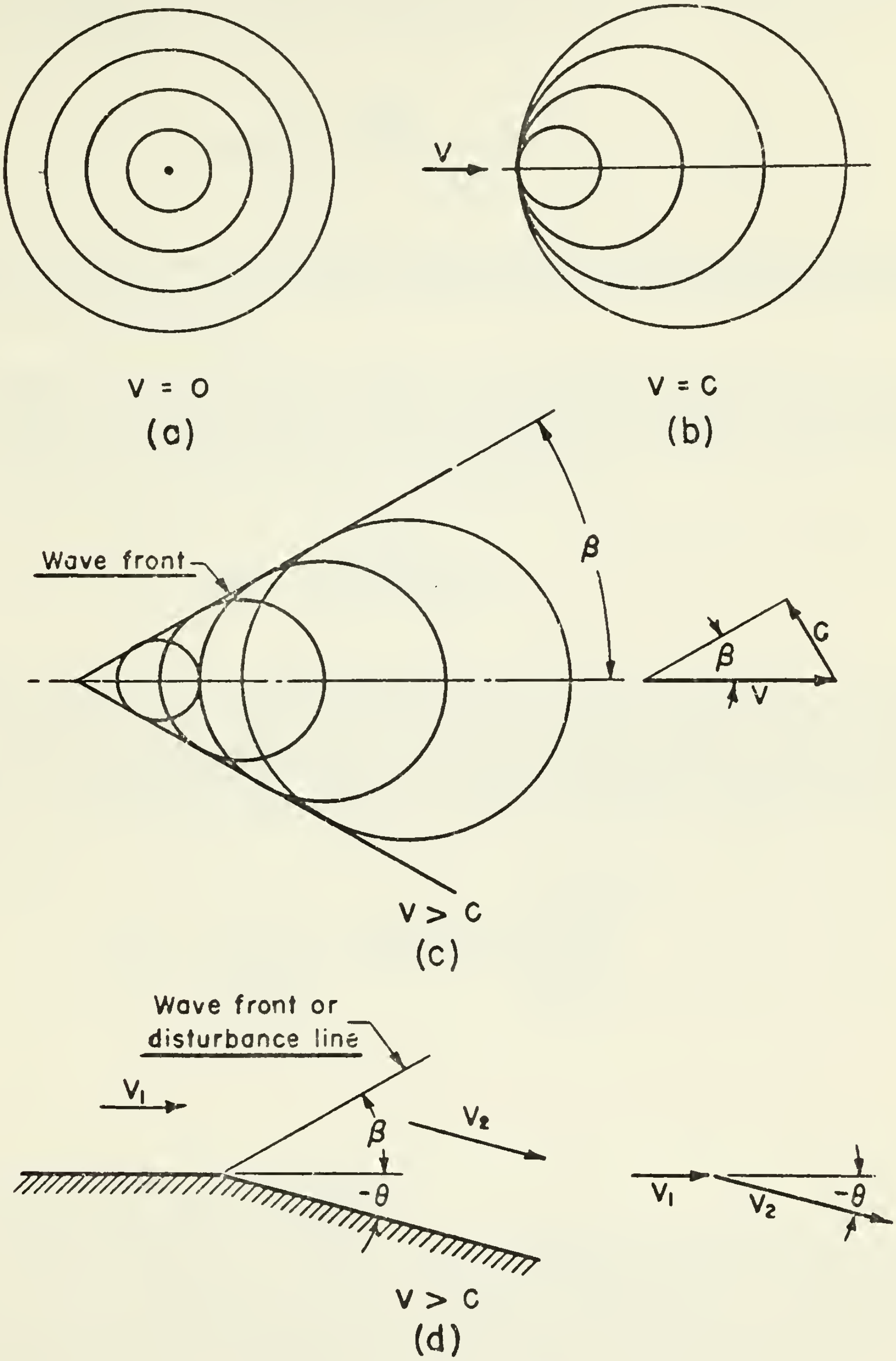
[2] VON KARMÁN, T. Eine praktische Anwendung der Analogie zwischen Ueberschallströmung in Gasen und überkritischer Strömung in offenen Gerinnen (A Practical Application of the Analogy between Supersonic Flow in Gases and Supercritical Flow in Open Channels). Zeitschrift Von Angewandte Mathematik Und Mechanik. Vol. 18, No. 1, pp.49-56. February 1938.

[3] The disturbance line is a standing wave in flows at supercritical velocities which originates at a disturbance or boundary discontinuity such as a bend in the wall of a channel.

[4] PREISWERK, E. Application of the Methods of Gas Dynamics to Water Flows with Free Surface, Part I. Flows with No Energy Dissipation. Institut für Aerodynamik Eidgenössische Technische Hochschule, Zürich. (Translated and published as Technical Memorandum No. 934, National Advisory Committee for Aeronautics, Washington, D. C., 75 pp. March 1940.

[5] HSU, EN-YUN. Characteristics of Supercritical Flow at a Gradual Open-channel Enlargement. Univ. Iowa Dept. Mech. and Hydrau. February 1946. (Unpublished M.S. thesis.)



WAVES CREATED BY DISTURBANCES IN FLOWING WATER

Figure 2

velocity the water has its minimum energy content, and the depth and velocity v_c are equal to their so-called critical values. At this point, then, $v_c = c = \sqrt{gd}$ and $\underline{F} = 1$. If the velocity is greater than the wave celerity, the continuously generated waves will travel downstream. The line drawn through the fronts of these waves in figure 2(c), page 5, is known as a wave front or disturbance line and lies at an angle to the original direction of flow. The magnitude of this angle, known as the wave angle β, is given by the equation

$$\sin\beta = \frac{c}{v} = \frac{\sqrt{gd}}{v} = \frac{1}{\sqrt{\underline{F}}}. \tag{3}$$

If the disturbance is continuous, such as a stick, a bridge pier, or the boundary discontinuity shown in figure 2(d), the waves originating at the disturbance will be continuously generated and the wave front will be a standing wave.

A fundamental conception is that changes in depth and in magnitude and direction of the velocity occur upon crossing the disturbance line. This conception is the basis for the derivation of the equations of depth and velocity change in transitions when the velocity of flow is greater than the critical. The differential equation for flow in transitions is

$$d\,\Theta = -\frac{1}{\bar{v}}\,\frac{\sqrt{(\bar{v})^2 - 1}}{\sqrt{1 - (\bar{v})^2/3}}\,d\bar{v} \qquad (4)[6]$$

where Θ is the angle between the original and the deflected velocity directions as shown in figure 2(d). When Θ is positive, the deflection results in an increase in depth; when Θ is negative, the deflection results in a decrease in depth. The latter is the case for expanding transitions. The quantity $\bar{v}$ is a nondimensional velocity defined as the ratio of the velocity of flow to the critical velocity. The relationship between $\bar{v}$ and other pertinent variables is given in equation (5)

$$\bar{v} = \frac{v}{v_c} = \frac{v}{\sqrt{2gH/3}} = \sqrt{3(1 - d/H)} = \sqrt{\frac{3\underline{F}}{\underline{F} + 2}} \tag{5}$$

where $H = \dfrac{v^2}{2g} + d.$

B. Graphical Computation of Depths in Transitions

The writer has prepared detailed instructions for the graphical solution of flow problems for flaring transitions[7] using Preiswerk's method. The space required to outline the detailed procedure is not warranted for this paper. In a few words, the procedure is to construct a characteristics diagram from which the magnitudes of the velocity and depth and the directions of the velocity and disturbance are determined. These values are transferred to a plan of the transition with the aid of a coordinate system based on the characteristics diagram.

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[6] It is suggested that those who wish to check the derivation of this equation refer to the thesis by Hsu, footnote 5, page 4.

[7] BLAISDELL, F. W. Graphical Construction of the Flow in Transitions by the Method of Characteristics (After Preiswerk). U. S. Dept. Agr., Soil Conserv. Serv., Minneapolis, Minn. (Unpublished. Paper No. MN-R-3-24, May 1944, Revised February 1946.)

C. Effect of Nonhydrostatic Pressure Distribution

The differential equation for flow in transitions has been derived on the assumption that hydrostatic pressure exists in the vertical. At initial Froude numbers $\underline{F}_1$ equal to or a little greater than one, there is a high rate of surface curvature in the vicinity of and within the transition. Under these conditions the vertical acceleration of the water assumes an appreciable magnitude and should not be neglected; in other words, the pressure in the vertical is not hydrostatic. Similar surface curvature is present if the width-depth ratio is small--that is, for relatively deep channels. When the surface curve is convex upwards, the pressure in the vertical is less than hydrostatic, and the stream does not spread as rapidly as assumed. When the surface curve is concave upwards, the pressure is greater than hydrostatic, and the stream spreads more rapidly than assumed. At the higher Froude numbers there is a low rate of surface curvature, and the pressure distribution should be much closer to the hydrostatic. There is also less discrepancy between the actual and the hydrostatic pressures if the stream is relatively wide.

D. Effect of Friction

Although the theoretical analysis assumes that the fluid is ideal or frictionless, friction loss always occurs if water is moving in a channel. If the Manning coefficient is used in the Chezy formula, the magnitude of this loss varies with the velocity squared and inversely with the 4/3 power of the hydraulic radius or, in the case of wide channels, inversely with the 4/3 power of the depth. Therefore, the greatest disagreement between theory and practice can be expected at the higher velocities and at the lesser depths.

V. EQUIPMENT AND TEST METHODS

The apparatus used for the transition tests is described below. Following this there is a description of the test procedure.

A. Test Platform

A general view of the apparatus is shown in figure 3, page 8. The superstructure consists of a 1/4-inch thick plate steel platform 10 feet long by 4 feet 9 inches wide bounded on the sides by 12-inch high steel channel walls on which are mounted the rails on which the point gage carriage travels. The platform and floor beams are connected by adjustable studs located, where possible, on 1-foot centers in both directions to permit the platform to be adjusted to an exact plane surface. The supports are arranged so that the slope of the platform can be varied at will from zero to 0.44 (sine of the slope angle). The point gage carriage can be seen in figure 4, page 9.

B. Entrance Nozzles

A very long length of open channel would be necessary to secure uniform flow approaching the transition. This length was not readily available, and considerable sacrifice in the flexibility of the apparatus would be required to provide it. Nozzles were therefore provided to give the desired shape of cross section and velocity of flow, and a short section of open channel was provided to insure typical open channel flow conditions at the entrance to the test section. That is, to insure that the nonhydrostatic pressure distribution mentioned in Section IV-C and the drop-down curve of the water surface resulting therefrom have an opportunity to develop fully. Since only flows symmetrical about the centerline were to be studied, the half-model technique was adopted; only halves of each nozzle and transition were tested.

The nozzles were ordinarily 3 feet long; the upper 2 feet were covered with a steel plate while the downstream foot was covered with a transparent plastic plate. An open channel section having a length equal to 5 times the depth was provided downstream from the nozzle exit.

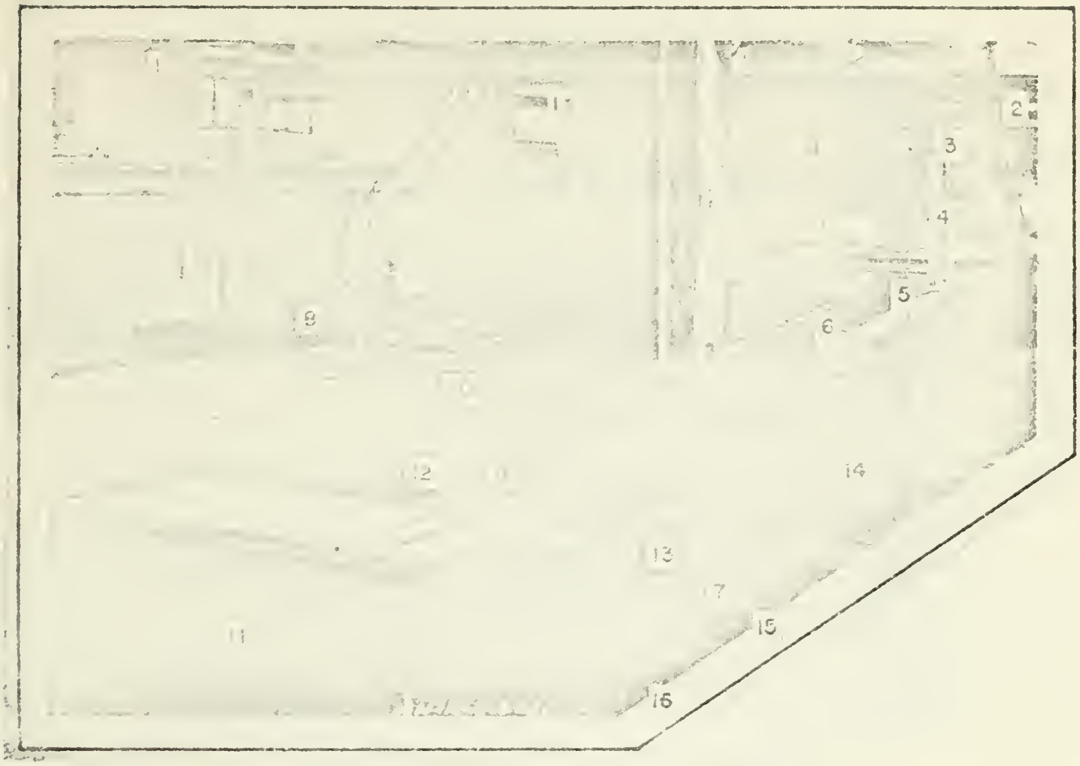

FIGURE 3.--Platform for testing transitions.

1. Supply line
2. Supply line (Venturi meter behind)
3. Control valve
4. Connecting hose
5. Approach pipe contains honeycomb
6. Nozzle
7. Slope gage
8. Booster pump
9. Rails
10. Point gage carriage
11. Waste collector
12. Platform
13. Floor beams
14. Girders
15. Adjustable support
16. Pivoted support

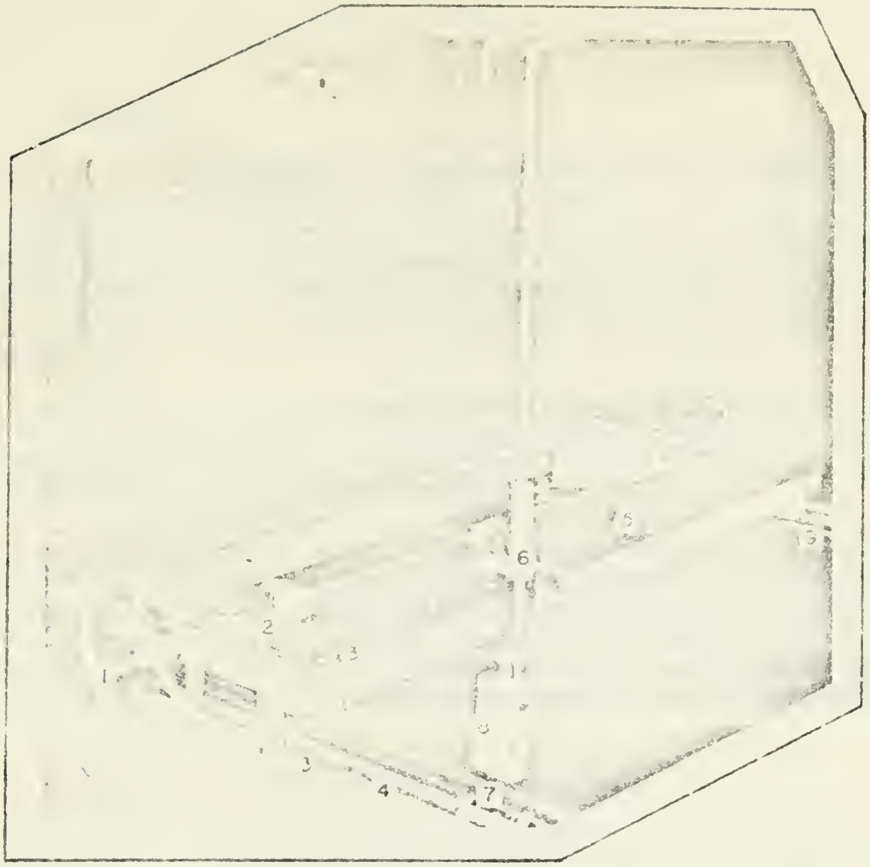

FIGURE 4.--Point gage carriage.

Handwheel geared to rack gives longitudinal movement. Tape on pulley provides transverse movement. Tapes and point gage permit determination of point location in three planes.

1. Longitudinal drive wheel
2. Transverse drive wheel
3. Pointers
4. Tapes
5. Transverse rail
6. Point gage
7. Longitudinal rails
8. Lamp
9. Rack

Bhoota[8] had found an open channel length of 3.5 times the depth to be satisfactory. Sizes of the nozzles tested and the corresponding width-depth ratios are presented in table 1, page 21. The width-depth ratios are those for full models.

C. Measurement of Discharge

All water used in the experiments was obtained from the main laboratory supply channel through the pipe system shown in figure 2, page 5. For many of the tests the laboratory gravity supply was used, but for a few tests it was necessary to resort to a booster pump to secure the desired discharge and velocity. All water used was measured by a calibrated Venturi meter. The accuracy of flow measurement was better than 1 percent.

D. Depth Measurements

The surface of the test platform was checked both transversely and longitudinally at 6-inch intervals. It was found that the usable portion was a plane to within ±0.001 foot with one exception. (Seventy-seven percent of the readings were within 0.0005 foot of a plane surface, 98 percent within 0.001 foot, and only one point deviated by 0.002 foot.)

All readings for determining the water surface contours were made to 0.001 foot. The point gage readings were read directly and the other readings estimated on steel tapes graduated to 0.01 foot. For low Froude numbers the accuracy of observation was about equal to the accuracy of making the readings. As the Froude number increased, it became progressively more difficult to determine the exact surface because of spray, and the accuracy of the measurements decreased. For this reason attempts to obtain surface readings for Froude numbers greater than 30 were abandoned in some instances.

VI. METHOD OF ANALYSIS

Considerable thought was given to methods of analysis that would achieve the greatest simplification in the final results and make their application, particularly to the design of SAF stilling basins, as easy as possible. To this end the greatest use possible was made of dimensionless parameters.

A. Plotting Surface Contours

The first step in analyzing the results of the tests was to prepare a plan of the transition for each Froude number at which tests were conducted. The relative depth contours d/d_1 were then plotted against x/b_1 and y/b_1, where x represents longitudinal distance, y transverse distance, and b_1 the width of the approach channel. The data for all width-depth ratios were plotted on a single plan because the theory indicates that the contours should coincide.

B. Determining Proportions of the Stream

While the method of plotting outlined above greatly simplified the presentation of the data, it was still in a form somewhat inconvenient to use. When planning transitions and SAF stilling basins, designers will need to know the mean depth of flow along the transition, the depth at the sidewall if it is greater than the mean depth, and the Froude number. The relative magnitudes of these variables were determined at several sections along the transition, and the results plotted. The use of the plots will be explained in Section VIII on "Summary Plot of Results," page 18.

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[8] BHOOTA, Baboobhai V. Characteristics of Supercritical Flow at an Abrupt Open-channel Enlargement. Univ. Iowa, Dept. Civil Engin. December 1942. (Unpublished Ph.D. thesis.)

VII. RESULTS OF TESTS

The results of the tests will be reported in four sections covering the required length of open approach channel, the effect of initial width-depth ratio, the effect of sidewall flare on the flow pattern, and the evaluation tests made on the sidewall flare found to be the most satisfactory.

A. Length of Open Approach Channel

It has been mentioned that the length of open channel approaching the transition was $5d_1$, and that Bhoota found $3.5d_1$ to be satisfactory. Hsu used the latter value. A length of open channel is necessary to simulate the nonhydrostatic pressure distribution that exists at the beginning of the transition.

The results of tests that show the effect of using a longer section of open channel or of eliminating the open channel are shown in figure 5, page 12. They are compared with those obtained with the standard length of open channel section, $5d_1$, and with those derived from the theoretical solution.

For one run summarized in figure 5(a) the plastic cover was removed from the nozzle making the approach channel $8d_1$ long. It is difficult to show the deviation from the standard condition because most of the contours coincide. It appears that an open channel length of $5d_1$ is sufficient to insure that the results obtained are typical of those that would be obtained if an open channel of infinite length had been used. This comparison is made at a low Froude number and low width-depth ratio, conditions for which the greatest deviation is to be expected.

The effect of covering the approach channel up to the beginning of the transition is shown in figures 5(b) and (c) for Froude numbers of 1 and 8 respectively. Covering the approach channel up to the beginning of the transition causes a downstream shift in the surface contours and agreement with the theoretical contours is generally worsened.

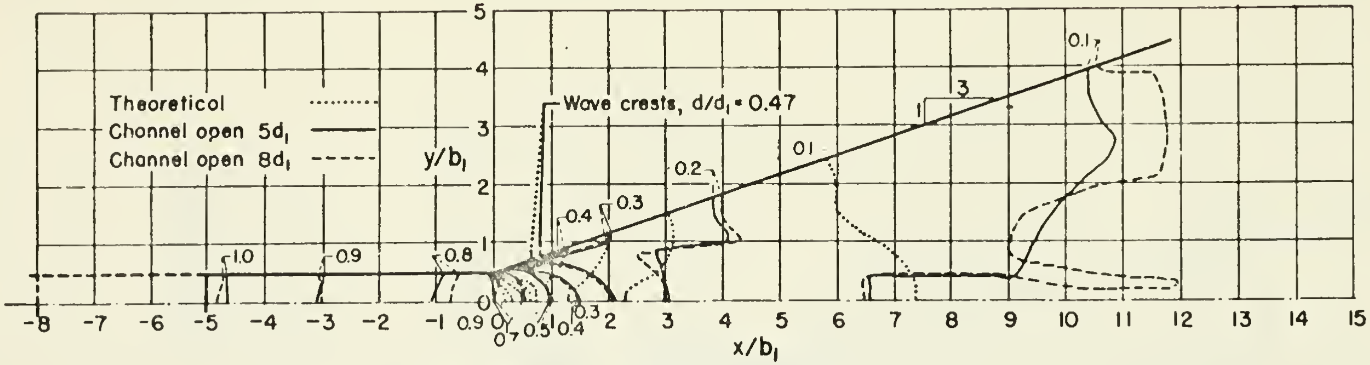
Since an open channel $5d_1$ long is sufficient to permit the establishment of flow conditions such as would exist if the approach channel had not been covered at all, this length of open channel was used at the approach to all transitions discussed in this report.

B. Effect of Width-Depth Ratio

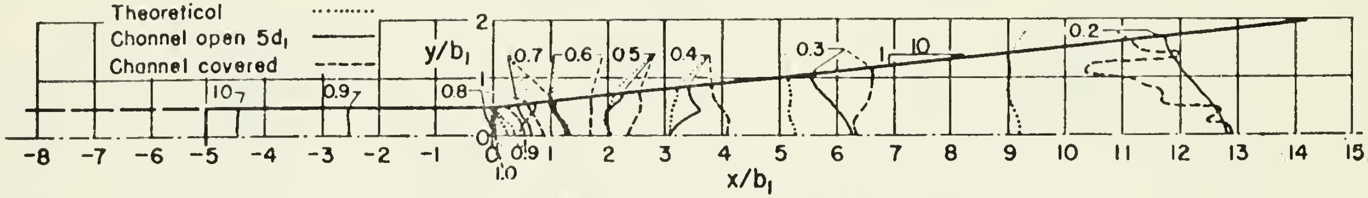
Theoretically, the surface contours in the transition should coincide for all width-depth ratios b_1/d_1 when the Froude numbers are identical and the coordinates in the plane of the platform are taken as x/b_1 and y/b_1. This was found to be the case as is shown in figure 6, page 13, which is typical of the results obtained for all Froude numbers. Many of the even-depth contours have been omitted from figure 6 to avoid confusion. The agreement between contours is felt to be reasonably good.

It will be noticed that the 1.0 contour does not begin at the disturbance as it should theoretically. This is a result of the nonhydrostatic pressure distribution in the vicinity of the transition and the draw-down curve. It will also be noticed that the discrepancy is less for the larger b_1/d_1 ratios (greater relative widths of entering stream). This is to be expected.

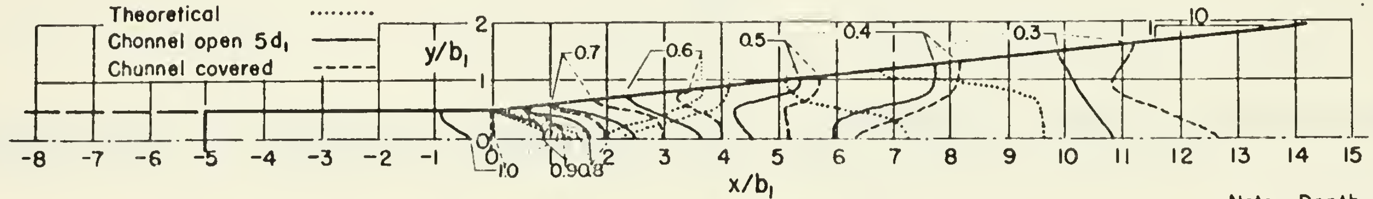
Views of the flow through transitions are presented in figure 7, page 14. The sharp drop-down at the beginning of the transition (the disturbance point) can be seen in the side views. The drop-down curves in the transition can be seen readily at both the lucite sidewall and the centerline. The platform floor is painted aluminum and the sidewall dark green. Their intersection provides a base line from which the depth at the centerline can be observed. The wave reflected from the sidewall can also be seen in many of the side and downstream views.



(a) $F_1 = 1$, $b_1/d_1 = 1$, $S = 0.015$



(b) $F_1 = 1$, $b_1/d_1 = 1$, $S = 0.015$

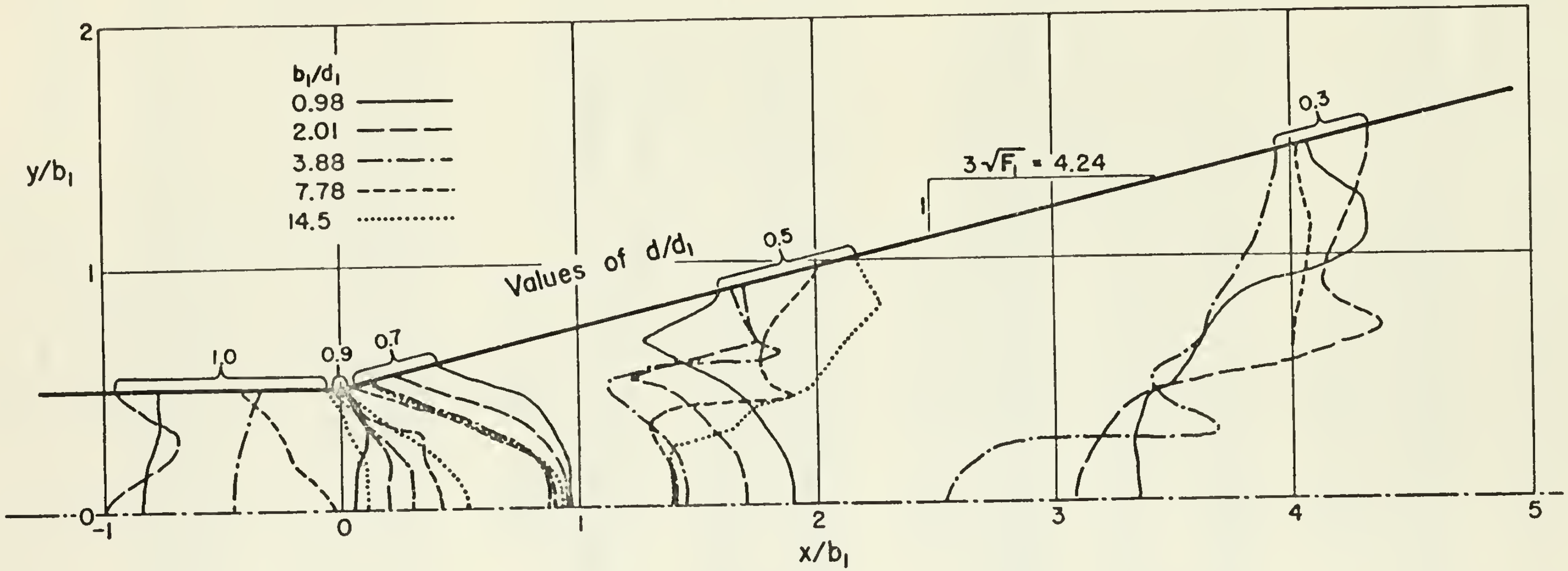


(c) $F_1 = 8$, $b_1/d_1 = 1$, $S = 0.116$

Note:- Depth contours are in terms of d/d_1.

EFFECT OF LENGTH OF OPEN CHANNEL SECTION ON DEPTH CONTOURS

Figure 5



COMPARISON OF SURFACE CONTOURS FOR DIFFERENT WIDTH-DEPTH RATIOS

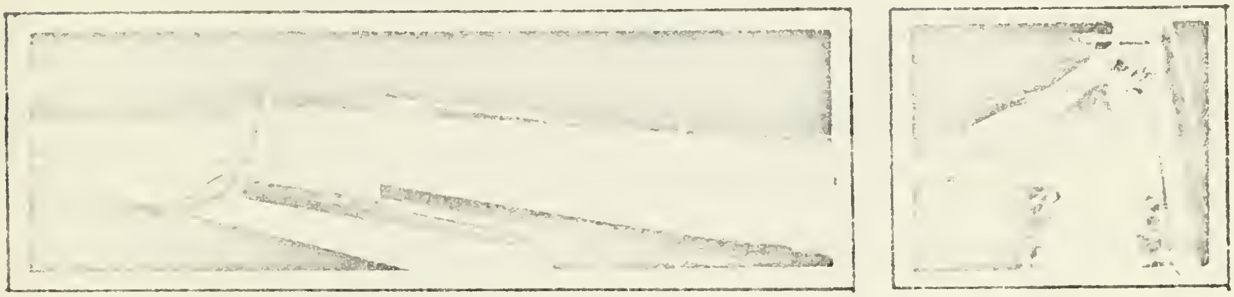
$F_1 = 2$ $S = 0.010$

MN-R-3-42, February 1947

Figure 6

(a) $b_1/d_1 = 0.96$

(b) $b_1/d_1 = 2.01$



(c) $b_1/d_1 = 3.88$

(d) $b_1/d_1 = 7.78$

FIGURE 7.--Flow in transition at several width-depth ratios.
$\underline{F} = 2$; $D = 4.24$; $S = 0.010$.

C. Maximum Permissible Sidewall Flare

The tests reported in this paper are confined to transitions having straight flaring sidewalls. Although this sidewall is probably the easiest to lay out in the field, its sharp intersection with the channel wall causes more disturbance to the flow than does a curving sidewall that intersects the channel wall at a tangent. Since the disturbance increases with the flare, tests were made to determine the maximum permissible sidewall divergence. Two principal considerations were kept in mind when the results of the tests were evaluated. The flare must be as rapid as possible in order that the desired flow depth be secured in the shortest possible distance, and the disturbances created must not be objectionable from a practical point of view. The criterion used was that the height of disturbance reflected from the sidewall near the beginning of the transition must not exceed the depth of flow at the transition centerline. However, at the downstream end of the transition the depth at the sidewall did exceed that at the centerline, as will be noted later in Section VIII, "Summary Plot of Results," page 18.

The results of the tests to determine the maximum permissible sidewall flare are presented in figure 8, page 16. They were made using a single nozzle in which $b_1/d_1 = 1$ on the assumption that any flare that proved satisfactory for the lowest b_1/d_1 ratio would be satisfactory for the higher valued ratios. Later tests confirmed the validity of this assumption. Figure 6, page 13, shows typical contours for the several b_1/d_1 ratios on which tests were conducted.

The results shown in figure 8 were obtained by analyzing each test and rating the results. The mathematical symbols shown indicate whether the sidewall flare was greater than, equal to, or less than that required to insure good flow conditions on the transition. The curve shown was drawn among the satisfactory divergences, dividing the excessive divergences from those considered to be less than the maximum permissible. This curve has the equation

$$D = 3\sqrt{F_1} \qquad (6)$$

where the divergence is given in terms of D longitudinal to 1 transverse. This is also the divergence suggested by Bhoota although the writer could find no evidence that Bhoota had ever made tests to check this equation. Woodward and Posey[9] state that the divergence angle should be less than the wave angle. In other words, they say that the divergence should be less than the square root of the Froude number, approximately, for small angles. The tests reported here show that the maximum rate of flare is too great and should not be used.

The divergence given by equation (6) was used in all subsequent tests and produced satisfactory flow conditions within the transition.

D. Tests at Wall Flare of 1 in $3\sqrt{F_1}$

Evaluation tests were made at flares of 1 in $3\sqrt{F_1}$. For these tests the platform was set on a 1 percent slope, just sufficient to provide drainage. The tests covered a width of the approaching stream ranging from $b_1/d_1 = 0.96$ to $b_1/d_1 = 14.5$ and Froude numbers ranging from 1 to 50.

Typical views of the flow in the transition are shown in figure 9, page 17. The decrease in the angle of the wave front as the Froude number increases will be noticed. It is apparent from the pictures that the wave does not increase in height as the Froude number increases. This is because the wall angle is inversely proportional to the square root of the Froude number. That the water surface increases in roughness with the Froude number is also apparent. This makes exact determination of the surface contours difficult at the higher Froude numbers.

The quantitative results of the tests will be discussed in the following section:

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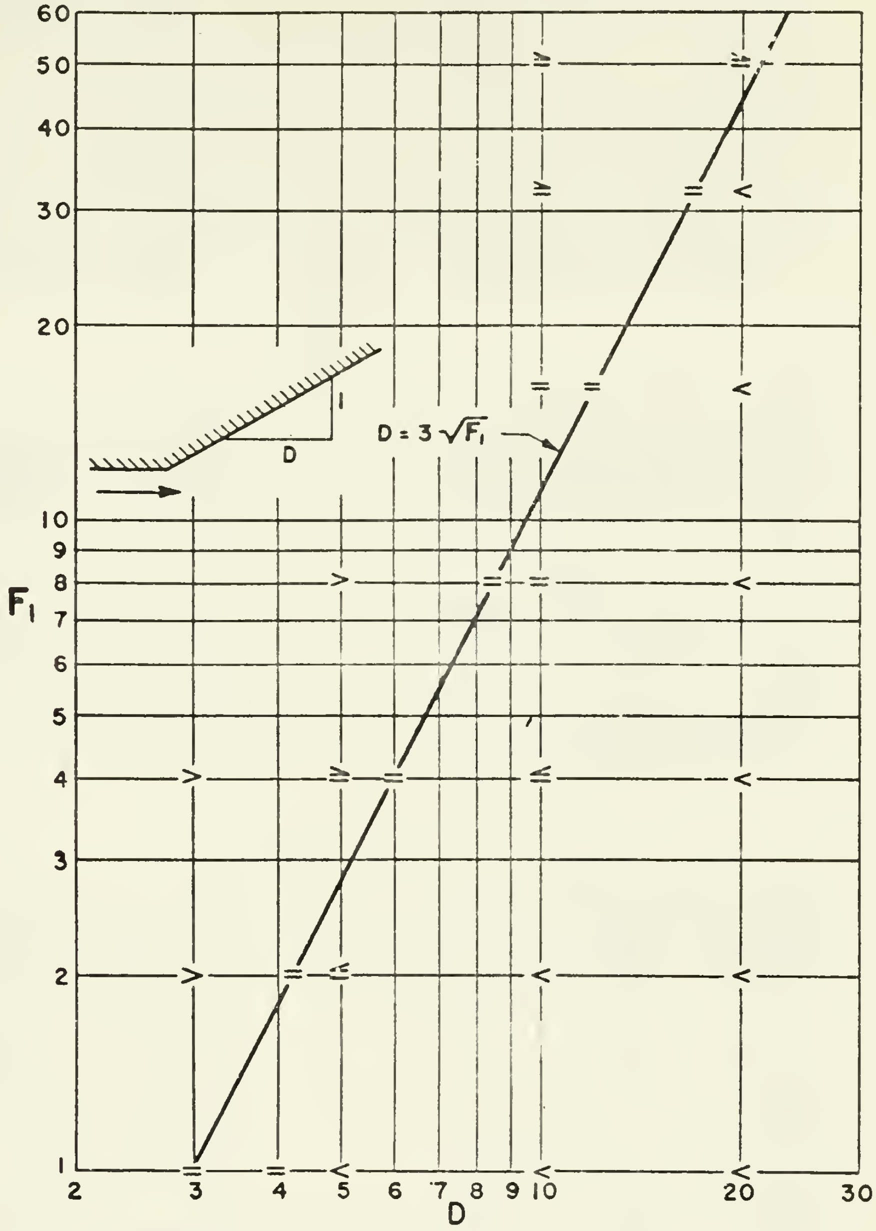
[9] WOODWARD, S. M., and POSEY, C. J. Hydraulics of Steady Flow in Open Channels. p. 125. New York. 1941.



Symbols:

< D satisfactory, could be less.

≤ D satisfactory, could be a little less.

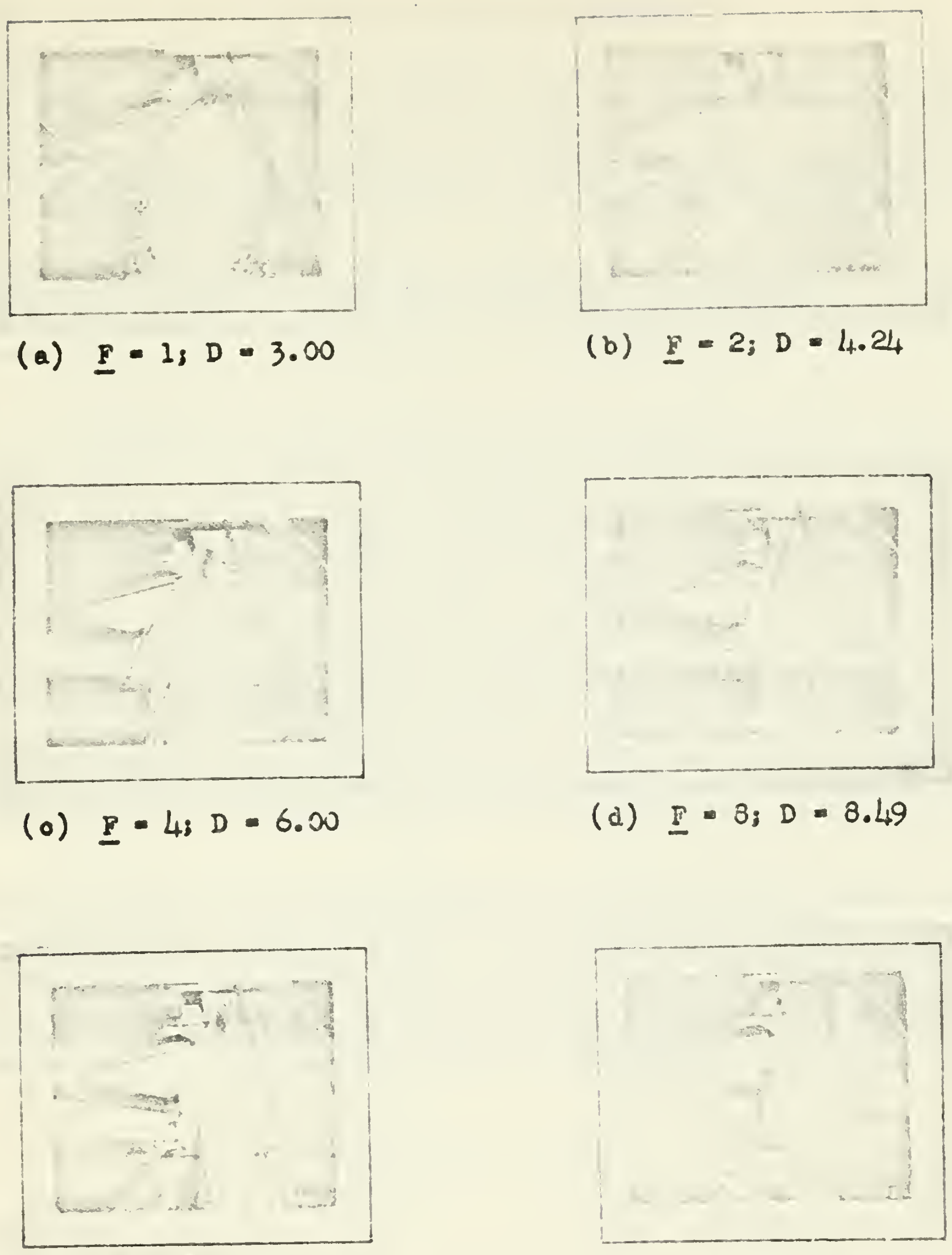
= D satisfactory, minimum permissible.

≥ D barely satisfactory, should be greater.

> D unsatisfactory, should be greater.

SIDEWALL DIVERGENCE

AS A FUNCTION OF THE FROUDE NUMBER



(a) $\underline{F} = 1$; $D = 3.00$

(b) $\underline{F} = 2$; $D = 4.24$

(c) $\underline{F} = 4$; $D = 6.00$

(d) $\underline{F} = 8$; $D = 8.49$

(e) $\underline{F} = 16$; $D = 12.00$

(f) $\underline{F} = 32$; $D = 16.98$

FIGURE 9.--Typical variation in flow pattern with Froude number.

$b_1/d_1 = 2.01$; $D = 3\sqrt{\underline{F}_1}$; $S = 0.010$.

VIII. SUMMARY PLOT OF RESULTS

The results of the evaluation tests made with a 1 percent platform slope and a sidewall divergence of $3\sqrt{\underline{F}_1}$ are summarized in figure 10, page 19. The plotted points are the averages for all width-depth ratios. These averages were obtained for all values of x/b_1 where the data for a sufficient number of cross sections were available to make this procedure feasible. The maximum x/b_1 values at which it was possible to obtain data varied inversely with b_1. For $b_1/d_1 = 1$ the maximum was about 12, while for $b_1/d_1 = 14.5$, it was only about 3.

The curves drawn in figure 10 are those suggested for design purposes. For the lowest Froude numbers the curves for the mean relative depths have been projected from $x/b_1 = 3$ or 4 to $x/b_1 = 6$. It is believed that the accuracy of the projected curves is on a par with that of their initial portions.

The mean relative areas of the wetted cross sections are plotted in figure 10(a). This plot may be used to determine the average area at each cross section and the average velocity of flow. It will be noticed that the relative area decreases with distance along the transition for the smaller Froude numbers. This is to be expected since the depth decreases, the depth energy is converted to kinetic energy, and the higher velocity requires a smaller area in accordance with the continuity equation. However, at the higher Froude numbers the area increases with distance. The reason for this is not definitely known. It might be ascribed to the frictional effects since the friction slope was many times the bed slope and the areas at all cross sections exceed the computed theoretical areas. Wilson,[10] in discussing his studies, has said that "frictional drag is insufficient to explain the observed differences between normal and actual depths." On the other hand, Hsu,[11] discussing the effect of slopes on surface contours stated that the single test made at the friction slope "evidences no appreciable effect on the surface contours in comparison with those obtained when the table was level." Additional tests are planned with the transition platform set on friction slopes. For the present, however, the available data do not permit definite conclusions to be drawn regarding the effects of friction on the cross-sectional areas.

The relative area divided by the relative width gives the mean relative depth. These values are plotted in figure 10(b). They may be used, in conjunction with figure 10(a), in computing the Froude number at each cross section. They may also be used in computing the required sidewall height. However, the depth at the sidewall sometimes exceeds the mean depth in the cross section, and extra sidewall height must be added to prevent overtopping. The expected excess of depth at the sidewall over the mean depth is given in figure 10(c). In view of the results presented in figure 10(c), it is suggested that the depth at the sidewall be assumed at

$$d_1\left(\frac{d}{d_1} + 0.2\right) = d + 0.2d_1,$$

but not greater than d_1.

Figure 10(d) was prepared to show the relative increase in the Froude number at any cross section and to assist in the design of stilling basins. If the initial Froude number is known, the local mean Froude number can be found by multiplying $\underline{F}_1$ by a value obtained from figure 10(b).

IX. USE OF TRANSITION WITH SAF STILLING BASIN

It was mentioned in Section III, "Reason for Tests," page 2, that the use of a transition would permit a smaller SAF stilling basin to be used and that the cost of building a transition

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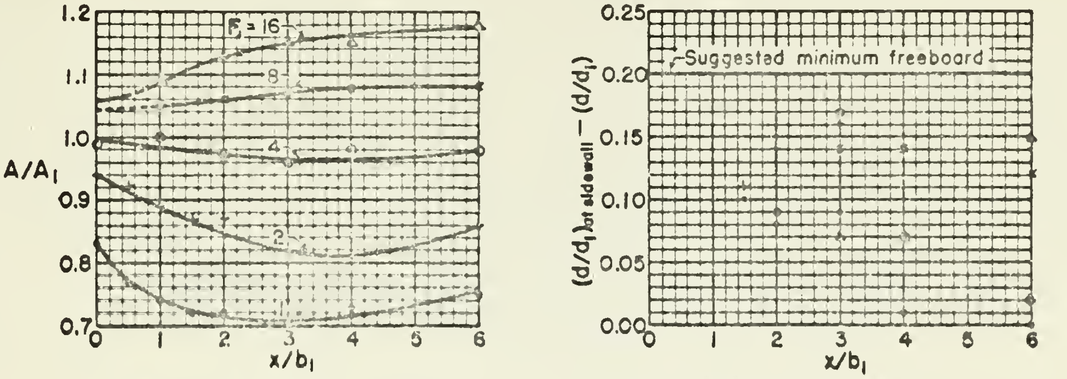
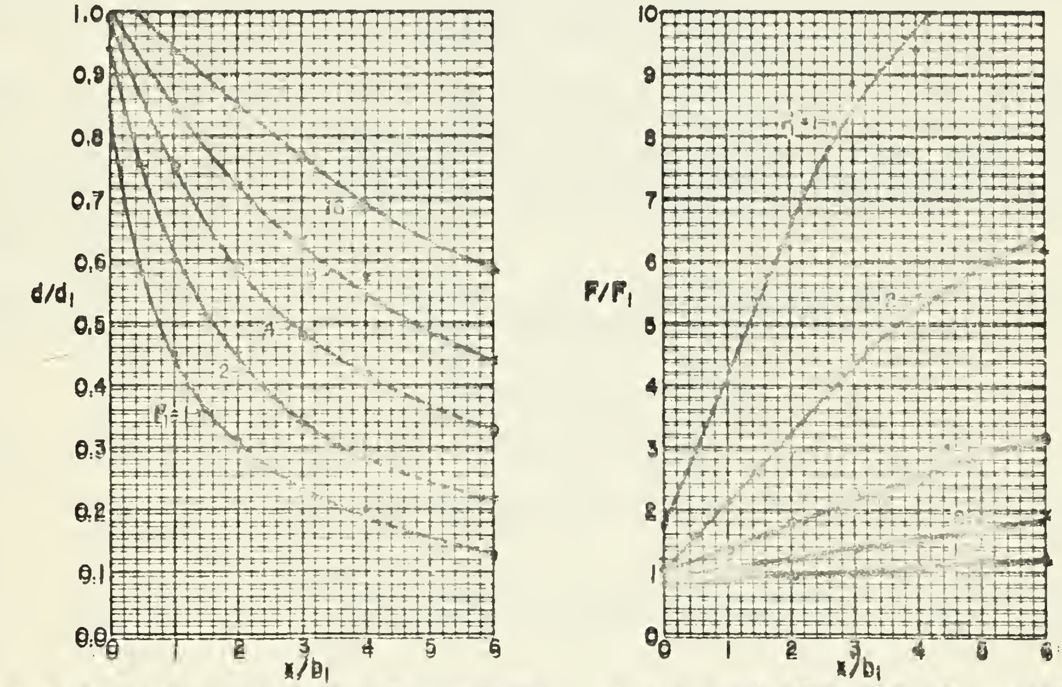
[10] WILSON, W. E. Flow Transitions in Rectangular Channels with Supercritical Velocities. Univ. Iowa Dept. Mech. and Hydrau. August 1940. (Unpublished Ph.D. thesis.)

[11] See footnote 5, page 4.



(a) MEAN RELATIVE AREA OF WETTED CROSS SECTION

(c) EXCESS OF RELATIVE DEPTH AT SIDE-WALL OVER MEAN RELATIVE DEPTH



(b) MEAN RELATIVE DEPTH IN CROSS SECTION

(d) MEAN RELATIVE FROUDE NUMBER

SUMMARY OF RESULTS

$D = 3\sqrt{F_1}$ $S = 0.010$

MN-R-3-44, February 1947

Figure 10

would be offset by savings in cost of the stilling basin. The curves of figure 10, page 19, permit an approximate quantitative comparison of the required sizes of stilling basin without and with a transition.

In order to make this comparison certain assumptions are necessary. These will not necessarily jibe with field conditions, but they will permit a comparison of the relative costs for the assumed conditions. In making the analysis it was assumed that the extra concrete required in the floor of the transition due to widening the channel approaching the stilling basin would be completely compensated for by the reduction in sidewall concrete because the sidewalls in the transition would be reduced in height. The use of a transition will also shorten the length of the approach channel required, and an additional saving will be obtained which was not considered in these computations. In the stilling basin proper the computed concrete volume includes only the floor, wall, and wingwall concrete; the variations in concrete required in the cutoff wall, end sill, and blocks were not considered. It should be noted that the use of a transition reduces the sidewall height and length and the floor length, and increases the floor width. Since the sidewall concrete is more expensive to place than the floor concrete, the monetary saving will actually be greater than is indicated; concrete volumes only were compared and not the actual cost of the concrete in place.

Estimated savings in the amount of excavation required were also determined. The original excavation required was assumed to be equal to the stilling basin width times its length times the tailwater depth. The excavation required when a transition is used was computed on the same basis. The savings in excavation is the difference between the two values.

The indicated savings in concrete and excavation were computed for three different b_1/d_1 ratios. They are $b_1/d_1 = 1$, such as might exist for a box culvert, possibly one of the drop-inlet type; $b_1/d_1 = 2$, such as might exist for a twin-barreled culvert; and $b_1/d_1 = 8$, such as might exist for a chute. For drop inlet spillways initial Froude numbers $\underline{F}_1$ in the range 3 to 8 may be expected. For chute spillways an initial Froude number of 16, the maximum for which transition design data are available, is low.

The results shown in **table 2, page 21,** are presented without comment except to note that the indicated savings are much greater than had been originally thought possible.

X. SUMMARY OF RESULTS

The results of the tests may be summarized as follows:

1. A length of open approach channel preceding the transition equal to $5d_1$ is satisfactory.
2. The relative depth contours for all values of b_1/d_1 may be combined through the use of the coordinates x/b_1 and y/b_1.
3. The maximum permissible sidewall divergence is 1 in $3\sqrt{\underline{F}_1}$ if cross waves of excessive height are to be avoided.
4. The curves of **figure 10** may be used to determine flow conditions within the transition.
5. The use of a transition in conjunction with the SAF stilling basin is economical.

TABLE 1.--*Dimensions of nozzles*

Nozzle dimension--inches					
Depth	4	3	2	1-1/2	1
Half-width	2	3	4	6	7-1/2
Width-depth ratio	1	2	4	8	15

TABLE 2.--*Effect of transition on SAF stilling basin size*

Trans-ition length	Froude No.	Comparison based on initial cross section					
		4 x 4 ft.		4 x 8 ft.		1 x 8 ft.	
		Percentage saving in					
$\frac{x}{b_1}$	F	Concrete	Excavation	Concrete	Excavation	Concrete	Excavation
0	1.0	—	—	—	—	—	—
1	4.1	5	-75	3	-43	-1	-20
2	6.5	27	-36	20	-20	4	-7
3	8.5	39	-15	28	-6	4	-1
4	9.7	50	9	37	14	10	20
6	11.9	61	38	48	40	17	41
0	2.0	—	—	—	—	—	—
1	4.2	13	-61	10	-30	2	-8
2	6.4	32	-27	26	-10	9	1
3	8.6	45	-5	37	6	15	13
4	10.4	53	9	44	16	18	21
6	12.9	60	20	49	24	13	27
0	4.0	—	—	—	—	—	—
1	5.6	15	-57	13	-27	6	-4
2	7.1	33	-25	29	-7	15	7
3	8.6	46	-3	41	8	21	17
4	10.1	52	3	45	12	22	18
6	12.8	61	17	53	22	25	26
0	8.0	—	—	—	—	—	—
1	8.6	17	-49	15	-22	9	-1
2	9.8	34	-25	31	-6	18	8
3	11.1	43	-8	40	5	25	15
4	12.3	51	2	46	12	29	20
6	14.8	62	19	56	25	35	30
0	16.0	—	—	—	—	—	—
1	14.4	15	-50	14	-23	8	-4
2	15.4	26	-33	24	-14	16	1
3	16.4	35	-23	33	-6	22	6
4	17.4	45	-7	42	5	30	14
6	19.4	55	6	52	15	35	21